Poole's Specificity Revised

Claus-Peter Wirth & Frieder Stolzenburg
wirth@logic.at, fstolzenburg@hs-harz.de
FB AI, Hochschule Harz, 38855 Wernigerode, Germany

Specificity of Explanations in Logic Programm.

- [Poole, 1985]: David L. Poole.
  On the Comparison of Theories: Preferring the Most Specific Explanation.
  IJCAI 9, pp. 144–147. Binary Specificity Relation
- [Simari & Loui, 1992]: Guillermo R. Simari, Ronald P. Loui.
  A Mathematical Treatment of Defeasible Reasoning and its Implementation.
  Artificial Intelligence **53**, pp. 125–157.
  Specificity Relation is an Ordering
- [Stolzenburg &al., 2003]: Frieder Stolzenburg, Alejandro J. García, Carlos I. Chesñevar, Guillermo R. Simari.
  Computing Generalized Specificity.
  J. Applied Non-Classical Logics **13**, pp. 87–113.

Theory $\mathfrak{T}_\Pi$ / Derivability $\vdash$

A *literal* L is an atom A or a negated atom $\neg A$.
The negation symbol "$\neg$" provides an interface to the application context, but is not active in the notion of derivability.

Let Π be a set of *rules* of the form $L \left[\Leftarrow L'_0 \wedge \ldots \wedge L'_n \right]$.

The theory $\mathfrak{T}_\Pi$ is inductively defined to contain

- all instances of literals from Π, and
- all literals L for which there is a conjunction C of literals from $\mathfrak{T}_\Pi$ such that $L \Leftarrow C$ is an instance of a rule in Π.

For $\mathfrak{L} \subseteq \mathfrak{T}_\Pi$, we also write $\Pi \vdash \mathfrak{L}$.

Specifications will always be given by 3 Sets:

- Π^{F} of literals — Facts of the concrete situation under consideration
- Π^{G} of rules — General rules that hold in all possible worlds
- Δ of rules — Defeasible / Default rules that hold in most situations

The set $\Pi := \Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}$ is the set of *strict* rules that — contrary to the defeasible rules — are

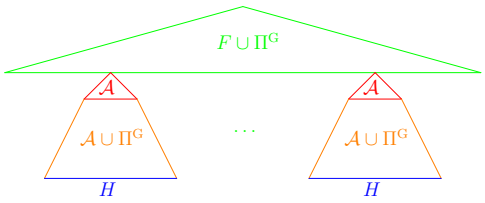
- considered to be safe and
- not doubted in the concrete situation.

Specificity [Quasi-] Orderings on Arguments

$(\mathcal{A}, L)$ is an *argument* if $\mathcal{A}$ is a set of ground instances of Δ, L is a literal, and $\mathcal{A} \vdash L$.

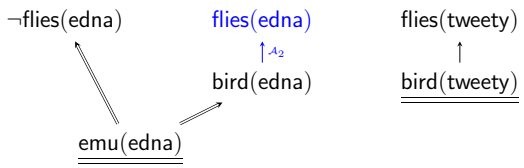
- Non-Transitive Specificity Relations:
  - $\lesssim_{\mathrm{P1}}$ David Poole's original
  - $\lesssim_{\mathrm{P2}}$ Guillermo R. Simari's minor correction of $\lesssim_{\mathrm{P1}}$
  - $\lesssim_{\mathrm{P3}}$ Our minor correction of $\lesssim_{\mathrm{P2}}$
- Transitive Specificity Relations (Quasi-Orderings!):
  - $\lesssim_{\mathrm{CP1}}$ Compared to $\lesssim_{\mathrm{P3}}$:
    – Reduction from $\mathfrak{T}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}} \cup \Delta}$ to $\mathfrak{T}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}}$.
    – Admission of Π^{F} after *completed* defeasible argumentations.
  - $\lesssim_{\mathrm{CP2}}$ Efficiency improvement of $\lesssim_{\mathrm{CP1}}$

Defeasible Parts of Derivation w.r.t. $(\Pi^{\mathrm{F}}, \Pi^{\mathrm{G}}, \Delta)$



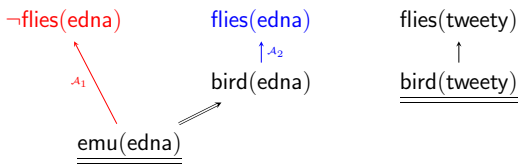
	$H \subseteq \ldots$	$F = \ldots$
P1, P2, P3	$\mathfrak{T}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}} \cup \Delta}$	H
CP1, CP2	$\mathfrak{T}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}}$	Π^{F}

Example 1 of [Poole, 1985]



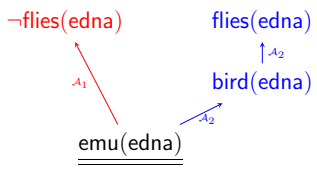
$\Pi^{\mathrm{F}} := \{\mathsf{bird}(\mathsf{tweety}), \mathsf{emu}(\mathsf{edna})\}$,
$\Pi^{\mathrm{G}} := \{\mathsf{bird}(x) \Leftarrow \mathsf{emu}(x),\ \ \neg\mathsf{flies}(x) \Leftarrow \mathsf{emu}(x)\}$,
$\Delta := \{\mathsf{flies}(x) \leftarrow \mathsf{bird}(x)\}$.
$\mathcal{A}_2 := \{\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{bird}(\mathsf{edna})\}$.
$(\emptyset, \neg\mathsf{flies}(\mathsf{edna})) <_{\mathrm{P1,P2,P3,CP1,CP2}} (\mathcal{A}_2, \mathsf{flies}(\mathsf{edna}))$.

Ex.2 of [Poole,1985]. Pref. of "More Concise"



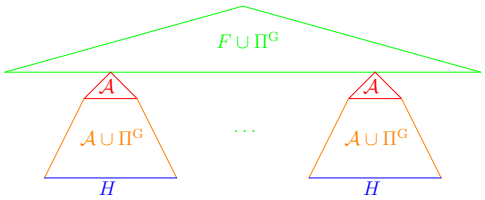
$\Pi^{\mathrm{F}} := \{\mathsf{bird}(\mathsf{tweety}), \mathsf{emu}(\mathsf{edna})\}$, $\Pi^{\mathrm{G}} := \{\mathsf{bird}(x) \Leftarrow \mathsf{emu}(x)\}$,
$\Delta := \{\mathsf{flies}(x) \leftarrow \mathsf{bird}(x),\ \ \neg\mathsf{flies}(x) \leftarrow \mathsf{emu}(x)\}$.
$\mathcal{A}_1 := \{\neg\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{emu}(\mathsf{edna})\}$.
$\mathcal{A}_2 := \{\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{bird}(\mathsf{edna})\}$.
$(\mathcal{A}_1, \neg\mathsf{flies}(\mathsf{edna})) <_{\mathrm{P1,P2,P3,CP1,CP2}} (\mathcal{A}_2, \mathsf{flies}(\mathsf{edna}))$.

Example 3 of [Poole, 1985] (renamed)



$\Pi^{\mathrm{F}} := \{\mathsf{emu}(\mathsf{edna})\}$, $\Pi^{\mathrm{G}} := \emptyset$
$\Delta := \{\mathsf{flies}(x) \leftarrow \mathsf{bird}(x),\ \ \neg\mathsf{flies}(x) \leftarrow \mathsf{emu}(x),\ \ \mathsf{bird}(x) \leftarrow \mathsf{emu}(x)\}$.
$\mathcal{A}_1 := \{\neg\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{emu}(\mathsf{edna})\}$.
$\mathcal{A}_2 := \{\mathsf{flies}(\mathsf{edna}) \leftarrow \mathsf{bird}(\mathsf{edna}),\ \ \mathsf{bird}(\mathsf{edna}) \leftarrow \mathsf{emu}(\mathsf{edna})\}$.
$(\mathcal{A}_1, \neg\mathsf{flies}(\mathsf{edna})) \approx_{\mathrm{CP1,CP2}} (\mathcal{A}_2, \mathsf{flies}(\mathsf{edna}))$.
$(\mathcal{A}_1, \neg\mathsf{flies}(\mathsf{edna})) <_{\mathrm{P1,P2,P3}} (\mathcal{A}_2, \mathsf{flies}(\mathsf{edna}))$.

Defeasible Parts of Derivation w.r.t. $(\Pi^{\mathrm{F}}, \Pi^{\mathrm{G}}, \Delta)$



	$H \subseteq \ldots$	$F = \ldots$
P1, P2, P3	$\mathfrak{X}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}} \cup \Delta}$	H
CP1, CP2	$\mathfrak{X}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}}$	Π^{F}

Computing Specificity Relations

(phase 1) Derive the literals that provide the basis for specificity considerations. CP1/2: $\mathfrak{X}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}}$. P1–3: $\mathfrak{X}_{\Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}} \cup \Delta}$.

(phase 2) On the basis of
- a subset H of the literals derived in phase 1,
- the first item $\mathcal{A}$ of a given argument $(\mathcal{A}, L)$, and
- the general rules Π^{G},

we derive a further set of literals $\mathfrak{l}$: $H \cup \mathcal{A} \cup \Pi^{\mathrm{G}} \vdash \mathfrak{l}$.

(phase 3) On the basis of $\mathfrak{l}$, the literal of the argument is derived: $\mathfrak{l} \cup \Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}} \vdash \{L\}$.
P1–3: phase 3 is empty: $\mathfrak{l} = \{L\}$.
CP1/2: It is admitted to use the facts from Π^{F} in phase 3, in addition to the general rules from Π^{G}.

Definition [Minimal] [Simplified] Activation Set

Let $\mathcal{A}$ be a set of ground instances of rules from Δ, and let L be a literal.

H is a *simplified activation set for* $(\mathcal{A}, L)$ if $L \in \mathfrak{X}_{H \cup \mathcal{A} \cup \Pi^{\mathrm{G}}}$.

H is an *activation set for* $(\mathcal{A}, L)$ if, for some $\mathfrak{l} \subseteq \mathfrak{X}_{H \cup \mathcal{A} \cup \Pi^{\mathrm{G}}}$, $L \in \mathfrak{X}_{\mathfrak{l} \cup \Pi^{\mathrm{F}} \cup \Pi^{\mathrm{G}}}$.

H is a *minimal* [*simplified*] *activation set for* $(\mathcal{A}, L)$ if H is an [simplified] activation set for $(\mathcal{A}, L)$, but no proper subset of H is an [simplified] activation set for $(\mathcal{A}, L)$.

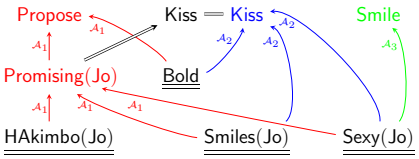
Definitions of $\lesssim_{\text{CP1}}$ and $\lesssim_{\text{P3}}$

$(\mathcal{A}_1, L_1) \lesssim_{\text{CP1}} (\mathcal{A}_2, L_2)$ if $(\mathcal{A}_1, L_1)$ and $(\mathcal{A}_2, L_2)$ are arguments, and we have

1. $L_1 \in \mathfrak{X}_{\Pi^{\text{F}} \cup \Pi^{\text{G}}}$ or
2. $L_2 \notin \mathfrak{X}_{\Pi^{\text{F}} \cup \Pi^{\text{G}}}$ and
   every $H \subseteq \mathfrak{X}_{\Pi^{\text{F}} \cup \Pi^{\text{G}}}$ that is an [minimal] activation set for $(\mathcal{A}_1, L_1)$
   is also an activation set for $(\mathcal{A}_2, L_2)$.

$(\mathcal{A}_1, L_1) \lesssim_{\text{P3}} (\mathcal{A}_2, L_2)$ if $(\mathcal{A}_1, L_1)$ and $(\mathcal{A}_2, L_2)$ are arguments,
$L_2 \in \mathfrak{X}_{\Pi^{\text{F}} \cup \Pi^{\text{G}}}$ implies $L_1 \in \mathfrak{X}_{\Pi^{\text{F}} \cup \Pi^{\text{G}}}$, and,
for every $H \subseteq \mathfrak{X}_{\Pi^{\text{F}} \cup \Pi^{\text{G}} \cup \Delta}$
that is a [minimal] simplified activation set for $(\mathcal{A}_1, L_1)$
but not a simplified activation set for $(\emptyset, L_1)$,
H is also a simplified activation set for $(\mathcal{A}_2, L_2)$.

Example Not Transitive. Pref. of "More Precise"



$$\mathcal{A}_1 := \left\{ \begin{array}{l} \text{Promising(Jo)} \leftarrow \text{HAkimbo(Jo)} \wedge \text{Smiles(Jo)} \wedge \text{Sexy(Jo)}, \\ \text{Propose} \leftarrow \text{Promising(Jo)} \wedge \text{Bold} \end{array} \right\},$$

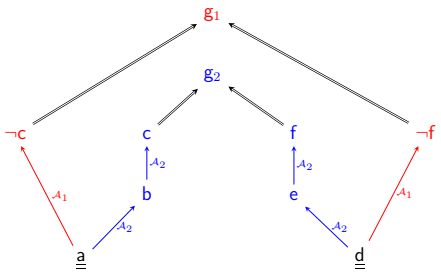
$\mathcal{A}_2 := \{\text{Kiss} \leftarrow \text{Bold} \wedge \text{Smiles(Jo)} \wedge \text{Sexy(Jo)}\}$,
$\mathcal{A}_3 := \{\text{Smile} \leftarrow \text{Sexy(Jo)}\}$. $\Pi^{\text{G}} := \{\text{Kiss} \Leftarrow \text{Promising}(G)\}$.
$\Pi^{\text{F}} := \{\text{Bold},\ \text{HAkimbo(Jo)},\ \text{Smiles(Jo)},\ \text{Sexy(Jo)}\}$.
$(\mathcal{A}_1, \text{Propose}) <_{\text{P1–3,CP1/2}} (\mathcal{A}_2, \text{Kiss}) <_{\text{P1–3,CP1/2}} (\mathcal{A}_3, \text{Smile})$, but
$(\mathcal{A}_1, \text{Propose}) <_{\text{CP1/2}} (\mathcal{A}_3, \text{Smile}) \not\lesssim_{\text{P1–3}} (\mathcal{A}_1, \text{Propose})$.

Conclusion: Novel Specificity Relations are . . .

- Transitive!
- Monotonic w.r.t. Conjunction!
- Even More Intuitive!
- Slightly More Efficient!
- More Comparing?

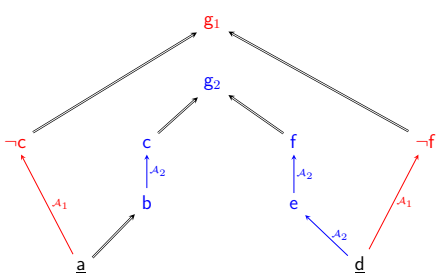
  Theorem: $\lesssim_{\text{P3}} \subseteq \lesssim_{\text{CP1}}$.
  Corollary: $\Delta_{\text{CP1}} \subseteq \Delta_{\text{P3}}$.
  But in general: $<_{\text{P3}} \not\subseteq <_{\text{CP1}}$. Luckily!
  (Otherwise monotonicity w.r.t. $\wedge$ would be lost.)

[Poole,1985,Example 6]: Monotonicity w.r.t. $\wedge$



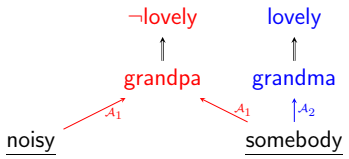
$(\mathcal{A}_1, \neg c) <_{\text{P1–3}} (\mathcal{A}_2, c)$, $(\mathcal{A}_1, \neg f) <_{\text{P1–3}} (\mathcal{A}_2, f)$, but $(\mathcal{A}_1, g_1) \Delta_{\text{P1–3}} (\mathcal{A}_2, g_2)$.
$(\mathcal{A}_1, \neg c) \approx_{\text{CP1/2}} (\mathcal{A}_2, c)$, $(\mathcal{A}_1, \neg f) \approx_{\text{CP1/2}} (\mathcal{A}_2, f)$, so $(\mathcal{A}_1, g_1) \approx_{\text{CP1/2}} (\mathcal{A}_2, g_2)$.

1st Variation of [Poole,1985,Example 6]



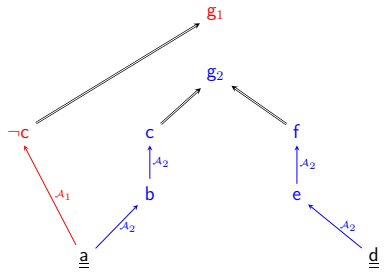
$(\mathcal{A}_1, \neg c) <_{\text{P1–3}} (\mathcal{A}_2, c)$, $(\mathcal{A}_1, \neg f) <_{\text{P1–3}} (\mathcal{A}_2, f)$, but $(\mathcal{A}_1, g_1) \Delta_{\text{P1–3}} (\mathcal{A}_2, g_2)$.
$(\mathcal{A}_1, \neg c) <_{\text{CP1/2}} (\mathcal{A}_2, c)$, $(\mathcal{A}_1, \neg f) \approx_{\text{CP1/2}} (\mathcal{A}_2, f)$, so $(\mathcal{A}_1, g_1) <_{\text{CP1/2}} (\mathcal{A}_2, g_2)$.

"More Precise"



$(\mathcal{A}_1, \neg \text{lovely}) <_{\text{P1–3}} (\mathcal{A}_2, \text{lovely})$.
$(\mathcal{A}_1, \neg \text{lovely}) <_{\text{CP1/2}} (\mathcal{A}_2, \text{lovely})$.

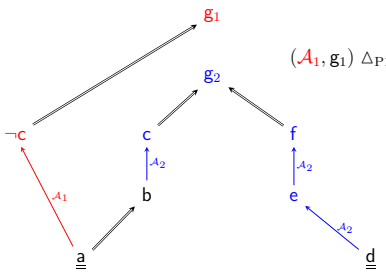
"More Precise", 2nd Var. of [Poole,1985,Ex. 6]



$(\mathcal{A}_1,\neg c) <_{\text{P1–3}} (\mathcal{A}_2,c)$, but $(\mathcal{A}_1,g_1) \vartriangle_{\text{P1–3}} (\mathcal{A}_2,g_2)$.

$(\mathcal{A}_1,\neg c) \approx_{\text{CP1/2}} (\mathcal{A}_2,c)$, but $(\mathcal{A}_1,g_1) >_{\text{CP1/2}} (\mathcal{A}_2,g_2)$, "more precise".

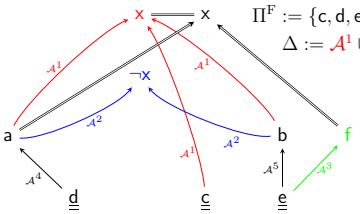
"Precise vs. Concise", 3rdVar.[Poole,1985,Ex. 6]



$(\mathcal{A}_1, g_1) \vartriangle_{\text{P1–3,CP1/2}} (\mathcal{A}_2, g_2)$

The conflict between a being "more concise" than b and b∧d being "more precise" than a is indeed irresolvable.

[Stolzenbg, 2003, Ex.11]: No Pruning for $\lesssim_{\text{P3}}$!

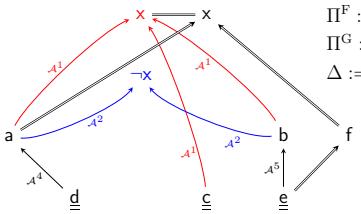


$\Pi^{\text{F}} := \{c, d, e\}$, $\Pi^{\text{G}} := \{x \Leftarrow a \wedge f\}$,
$\Delta := \mathcal{A}^1 \cup \mathcal{A}^2 \cup \mathcal{A}^3 \cup \mathcal{A}^4 \cup \mathcal{A}^5$.

$\mathcal{A}^1 := \{x \leftarrow a \wedge b \wedge c\}$.
$\mathcal{A}^2 := \{\neg x \leftarrow a \wedge b\}$.
$\mathcal{A}^3 := \{f \leftarrow e\}$.
$\mathcal{A}^4 := \{a \leftarrow d\}$.
$\mathcal{A}^5 := \{b \leftarrow e\}$.

$(\mathcal{A}^1 \cup \mathcal{A}^4 \cup \mathcal{A}^5, x) <_{\text{CP1/2}} (\mathcal{A}^2 \cup \mathcal{A}^4 \cup \mathcal{A}^5, \neg x) \approx_{\text{CP1/2}} (\mathcal{A}^3 \cup \mathcal{A}^4, x)$. All $\lesssim_{\text{P1–3}}$-incomparable! For $(\mathcal{A}^1 \cup \mathcal{A}^4 \cup \mathcal{A}^5, x) \not\lesssim_{\text{P3}} (\mathcal{A}^2 \cup \mathcal{A}^4 \cup \mathcal{A}^5, \neg x)$ we have to consider (implicitly via $\{d, f\} \subseteq \mathfrak{X}_{\Pi^{\text{F}} \cup \Pi^{\text{G}} \cup \Delta}$) the defeasible rule of $\mathcal{A}^3$, which is not part of any of the two arguments under comparison. No pruning possible for $\lesssim_{\text{P1–3}}$.

Variation of [Stolzenbg, 2003, Ex.11]

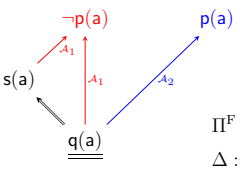


$\Pi^{\text{F}} := \{c, d, e\}$,
$\Pi^{\text{G}} := \{x \Leftarrow a \wedge f,\ f \Leftarrow e\}$,
$\Delta := \mathcal{A}^1 \cup \mathcal{A}^2 \cup \mathcal{A}^4 \cup \mathcal{A}^5$.

$\mathcal{A}^1 := \{x \leftarrow a \wedge b \wedge c\}$.
$\mathcal{A}^2 := \{\neg x \leftarrow a \wedge b\}$.
$\mathcal{A}^4 := \{a \leftarrow d\}$.
$\mathcal{A}^5 := \{b \leftarrow e\}$.

$(\mathcal{A}^4, x) \approx_{\text{CP1/2}} (\mathcal{A}^1 \cup \mathcal{A}^4 \cup \mathcal{A}^5, x) >_{\text{CP1/2}} (\mathcal{A}^2 \cup \mathcal{A}^4 \cup \mathcal{A}^5, \neg x)$.
Makes sense because d∧e is more precise (specific) than d.
c∧d∧e is irrelevant because approach is model-theoretic.
$(\mathcal{A}^4, x) <_{\text{P1–3}} (\mathcal{A}^1 \cup \mathcal{A}^4 \cup \mathcal{A}^5, x) \vartriangle_{\text{P1–3}} (\mathcal{A}^2 \cup \mathcal{A}^4 \cup \mathcal{A}^5, \neg x) \vartriangle_{\text{P1–3}}$
(Bullshit!) $(\mathcal{A}^4, x)$.

[Stolzenburg, 2003, Ex. p.95]: Global Effect!



$\Pi^{\text{F}} := \{q(a)\}$, $\Pi^{\text{G}} := \{s(x) \Leftarrow q(x)\}$,
$\Delta := \{p(x) \leftarrow q(x), \neg p(x) \leftarrow q(x) \wedge s(x)\}$.
$\mathcal{A}_1 := \{\neg p(a) \leftarrow q(a) \wedge s(a)\}$.
$\mathcal{A}_2 := \{p(a) \leftarrow q(a)\}$.

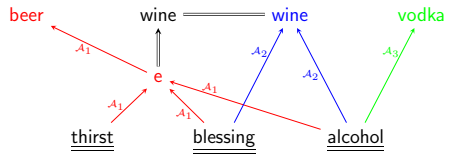
$(\mathcal{A}_1, \neg p(a)) \approx_{\text{P1–3,CP1/2}} (\mathcal{A}_2, p(a))$.

$\lesssim_{\text{CP1}}$ vs. $\lesssim_{\text{CP2}}$

$\Pi^{\text{F}} := \{\text{thirst}, \text{danger}\}$,
$\Pi^{\text{G}} := \emptyset$, $\Delta := \mathcal{A}_1 \cup \mathcal{A}_3$,
$\mathcal{A}_1 := \{\text{drink} \leftarrow \text{thirst}\}$,
$\mathcal{A}_2 := \{\text{alarm} \leftarrow \text{danger}\}$.
$\mathcal{A}_3 := \mathcal{A}_2 \cup \{\text{danger} \leftarrow \text{thirst}\}$.

$(\mathcal{A}_2, \text{alarm}) <_{\text{CP1}} (\mathcal{A}_3, \text{alarm}) \approx_{\text{CP2}} (\mathcal{A}_2, \text{alarm})$
$(\mathcal{A}_1, \text{drink}) <_{\text{CP1}} (\mathcal{A}_3, \text{alarm}) \vartriangle_{\text{CP2}} (\mathcal{A}_1, \text{drink})$
$(\mathcal{A}_1, \text{drink}) \vartriangle_{\text{CP1}} (\mathcal{A}_2, \text{alarm}) \vartriangle_{\text{CP2}} (\mathcal{A}_1, \text{drink})$

Example Not Transitive. Pref. of "More Precise"



$\mathcal{A}_1 := \{\mathsf{e} \leftarrow \mathsf{alcohol} \wedge \mathsf{blessing} \wedge \mathsf{thirst},\ \ \mathsf{beer} \leftarrow \mathsf{e}\}$,
$\mathcal{A}_2 := \{\mathsf{wine} \leftarrow \mathsf{alcohol} \wedge \mathsf{blessing}\}$,
$\mathcal{A}_3 := \{\mathsf{vodka} \leftarrow \mathsf{alcohol}\}$, $\Delta := \mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{A}_3$,
$\Pi^{\mathrm{F}} := \{\mathsf{alcohol}, \mathsf{blessing}, \mathsf{thirst}\}$, $\Pi^{\mathrm{G}} := \{\mathsf{wine} \Leftarrow \mathsf{e}\}$,
$(\mathcal{A}_1, \mathsf{beer}) <_{\mathrm{P1\text{–}3,CP1/2}} (\mathcal{A}_2, \mathsf{wine}) <_{\mathrm{P1\text{–}3,CP1/2}} (\mathcal{A}_3, \mathsf{vodka})$, but
$(\mathcal{A}_1, \mathsf{beer}) <_{\mathrm{CP1/2}} (\mathcal{A}_3, \mathsf{vodka}) \not\gtrsim_{\mathrm{P1\text{–}3}} (\mathcal{A}_1, \mathsf{beer})$.

All you need is Quasi-Ordering

A *quasi-ordering* is a reflexive transitive relation.
An *(irreflexive) ordering* is an irreflexive transitive relation.
A *reflexive ordering* (also called: "partial ordering") is an anti-symmetric quasi-ordering.
An *equivalence* is a symmetric quasi-ordering.

We will use several binary relations $\lesssim_N$ comparing arguments according to their specificity.

Corollary 1 *If $\lesssim_N$ is a quasi-ordering, then*
its equivalence $\approx_N$ is an equivalence,
its ordering $<_N$ is an ordering, and
its reflexive ordering $\leq_N$ is a reflexive ordering.

Abstract Specificity Orderings

For any relation written as $\lesssim_N$
("being more or equivalently specific w.r.t. N"),
we define:

$$\gtrsim_N := \{\,(X,Y) \mid Y \lesssim_N X\,\} \text{("less or equivalently specific")},$$
$$\approx_N := \lesssim_N \cap \gtrsim_N \text{("equivalently specific")},$$
$$<_N := \lesssim_N \setminus \gtrsim_N \text{("properly more specific")},$$
$$\leq_N := <_N \cup \{\,(X,X) \mid X \text{ is an argument}\,\} \text{("more specific or equal")},$$
$$\triangle_N := \left\{ (X,Y) \;\middle|\; \begin{array}{l} X, Y \text{ are arguments with} \\ X \not\lesssim_N Y \text{ and } X \not\gtrsim_N Y \end{array} \right\} \text{("incomparable w.r.t. specificity")}.$$

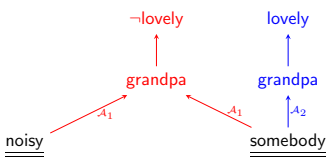
Efficiency Considerations

- Let H be a set of hypotheses. Then, roughly speaking, $(\mathcal{A}_1, L_1) \lesssim (\mathcal{A}_2, L_2)$ if every activation set H for $(\mathcal{A}_1, L_1)$ is also one for $(\mathcal{A}_2, L_2)$.
- Naïve procedure enumerates possible activation sets, which is exponential in the number of possible hypothesis literals.
- Clearly, the effort for computing $\lesssim_{\mathrm{CP1}}$ is lower than that of $\lesssim_{\mathrm{P3}}$ because of $\mathfrak{T}_\Pi \subseteq \mathfrak{T}_{\Pi\cup\Delta}$.
- While checking $\lesssim_{\mathrm{Px}}$, attention cannot be restricted to derivations which make use only of defeasible rules given in the arguments. Therefore, [Stolzenburg&al, 2003] introduce pruning derivation trees.

Path Characterization for Specificity

- $(\mathcal{A}_1, L_1) \leq (\mathcal{A}_2, L_2)$ if $(\mathcal{A}_1, L_1)$ and $(\mathcal{A}_2, L_2)$ are two arguments in the given specification and for each derivation tree T_1 for L_1 there is a derivation tree T_2 for L_2 such that $T_1 \trianglelefteq T_2$.
- Let T_1 and T_2 be derivation trees. Then, $T_1 \trianglelefteq T_2$ if for each $t_2 \in \mathsf{Paths}(T_2)$ there is a path $t_1 \in \mathsf{Paths}(T_1)$ such that $t_1 \subseteq t_2$ (omitting the root nodes).
- If the arguments involved in the comparison correspond to exactly one and-tree, then $\lesssim_{\mathrm{P2}}$ coincides with the path characterization ($\leq$ and $\trianglelefteq$). Cf. Example 1 of [Poole, 1985].
- Two and-trees can be compared efficiently w.r.t. $\trianglelefteq$. It requires pairwise comparison of all nodes in the trees for each path. Hence, the respective complexity is polynomial in the size of the derivation trees.

Path vs. Argument Characterization



- Poole's specificity: $(\mathcal{A}_1, \neg\mathsf{lovely}) <_{\mathrm{P1\text{–}3}} (\mathcal{A}_2, \mathsf{lovely})$
- Corrected version: $(\mathcal{A}_1, \neg\mathsf{lovely}) <_{\mathrm{CP1/2}} (\mathcal{A}_2, \mathsf{lovely})$
- Path characterization: $(\mathcal{A}_1, \neg\mathsf{lovely}) \leq (\mathcal{A}_2, \mathsf{lovely})$
  $\big\{\{\mathsf{noisy}, \mathsf{grandpa}\}, \{\mathsf{someb.}, \mathsf{grandpa}\}\big\}$ $\big\{\{\mathsf{someb.}, \mathsf{grandpa}\}\big\}$
- Argument sets: $(\mathcal{A}_1, \neg\mathsf{lovely}) \sqsubseteq (\mathcal{A}_2, \mathsf{lovely})$
  $\big\{\{\mathsf{noisy}, \mathsf{somebody}\}, \{\mathsf{grandpa}\}\big\}$ $\big\{\{\mathsf{somebody}\}, \{\mathsf{grandpa}\}\big\}$

Back to the Arguments

- Characterization dual to $\trianglelefteq$ based on the rules in the arguments:
- Let $(\mathcal{A}_1, L_1)$ and $(\mathcal{A}_2, L_2)$ be two arguments and R_i $(i = 1, 2)$ be the set of (strict and defeasible) rule bodies used in the respective proofs. We define: $R_1 \sqsubseteq R_2$ if for all $r_1 \in R_1$ there exists an $r_2 \in R_2$ such that $r_2 \subseteq r_1$.
- Simplified version: Interpret $\sqsubseteq$ simply as subset (w.r.t. complete rule sets).
- This is close to the notion *more conservative than* [Besnard&Hunter, 2001]: $\mathcal{A}_1 \subseteq \mathcal{A}_2$ and $L_2 \vdash L_1$.
- Checking $\subseteq$ on ground literal sets can be done efficiently (NP-complete for general literals).

Further Conclusions

- Computing $\lesssim_{\mathrm{CPx}}$ can be done by a modified SLD-resolution procedure, but has to enumerate all possible derivations for each query.
- Path ($\trianglelefteq$) and argument ($\sqsubseteq$) characterizations can be computed efficiently. However, they coincide with specificity notion only in special cases (e.g. no strict rules).
- Further investigation is required . . .
- Thank you very much for your attention!