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Some simple rippling examples

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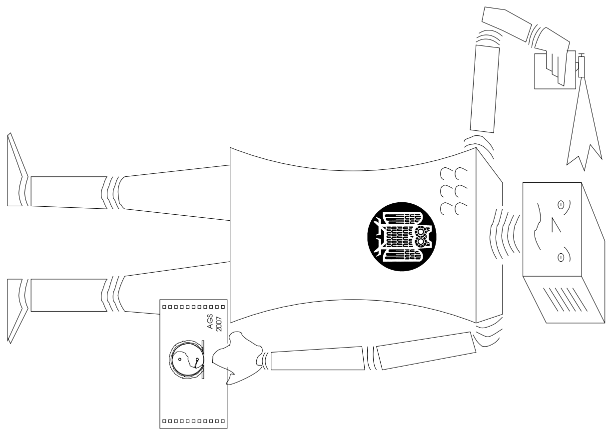

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Abstract

Keywords:

1 Simple Examples

Example 1.1

Given: $\mathsf{rev}(\mathsf{append}(k, l)) = \mathsf{append}(\mathsf{rev}(l), \mathsf{rev}(k))$.
Note that all variables in this goal are sinks!

Goal: $\mathsf{rev}^s(\mathsf{append}^s(\mathsf{cons}^\uparrow(x, k^s), l)) = \mathsf{append}^s(\mathsf{rev}^s(l), \mathsf{rev}^s(\mathsf{cons}^\uparrow(x, k^s)))$.
This goal is suggested by recursion analysis.

Wave-Rules:

$$
\begin{array}{lrcl}
(1) & \mathsf{rev}^s(\mathsf{cons}^\uparrow(x, l^s)) & = & \mathsf{append}^\uparrow(\mathsf{rev}^s(l^s), \mathsf{cons}(x, \mathsf{nil})) \\
(2) & \mathsf{append}^s(\mathsf{cons}^\uparrow(x, k^s), l^s) & = & \mathsf{cons}^\uparrow(x, \mathsf{append}^s(k^s, l^s)) \\
(3) & \mathsf{append}^s(h^s, \mathsf{append}^\uparrow(k^s, l)) & = & \mathsf{append}^\uparrow(\mathsf{append}^s(h^s, k^s), l)
\end{array}
$$

Note that, when a wave-rule is applied, only variables marked as skeleton may be substituted by terms with a skeleton marker!

Reduction of left-hand side of goal:

$$\mathsf{rev}^s(\mathsf{append}^s(\mathsf{cons}^\uparrow(x, k^s), l))$$

Wave-rule (2) via $\{x \mapsto x,\ \ k^s \mapsto k^s,\ \ l^s \mapsto l\}$:

$$\mathsf{rev}^s(\mathsf{cons}^\uparrow(x, \mathsf{append}^s(k^s, l)))$$

Wave-rule (1) via $\{x \mapsto x,\ \ l^s \mapsto \mathsf{append}^s(k^s, l)\}$:

$$\mathsf{append}^\uparrow(\mathsf{rev}^s(\mathsf{append}^s(k^s, l)), \mathsf{cons}(x, \mathsf{nil}))$$

Fully rippled! Cross fertilization via $\{k \mapsto k^s,\ \ l \mapsto l\}$ and removal of annotaton

$$\mathsf{append}(\mathsf{append}(\mathsf{rev}(l), \mathsf{rev}(k)), \mathsf{cons}(x, \mathsf{nil}))$$

Reduction of right-hand side of goal:

$$\mathsf{append}^s(\mathsf{rev}^s(l), \mathsf{rev}^s(\mathsf{cons}^\uparrow(x, k^s)))$$

Wave-rule (1) via $\{x \mapsto x,\ \ l^s \mapsto k^s\}$:

$$\mathsf{append}^s(\mathsf{rev}^s(l), \mathsf{append}^\uparrow(\mathsf{rev}^s(k^s), \mathsf{cons}(x, \mathsf{nil})))$$

Wave-rule (3) via $\{h^s \mapsto \mathsf{rev}^s(l),\ \ k^s \mapsto \mathsf{rev}^s(k^s),\ \ l \mapsto \mathsf{cons}(x, \mathsf{nil})\}$:

$$\mathsf{append}^\uparrow(\mathsf{append}^s(\mathsf{rev}^s(l), \mathsf{rev}^s(k^s)), \mathsf{cons}(x, \mathsf{nil}))$$

Fully rippled! After removal of annotation, this is equal to reduced form of left-hand side.

Example 1.2

Given: $\neg\mathsf{Even}(x),\ \neg\mathsf{Even}(y),\ \mathsf{Even}(x+y).$

Note that all variables in this goal are sinks!

Goal: $\neg\mathsf{Even}^s(\mathsf{s}^{\uparrow}(\mathsf{s}^{\uparrow}(x^s))),\ \neg\mathsf{Even}(y),\ \mathsf{Even}^s(\mathsf{s}^{\uparrow}(\mathsf{s}^{\uparrow}(x^s)) +^s y).$

This goal is suggested by recursion analysis.

Wave-Rules:

$$\begin{array}{lrcl} (4) & \mathsf{s}^{\uparrow}(x^s) +^s y^s & = & \mathsf{s}^{\uparrow}(x^s +^s y^s) \\ (5) & \mathsf{Even}^s(\mathsf{s}^{\uparrow}(\mathsf{s}^{\uparrow}(x^s))) & = & \mathsf{Even}^s(x^s) \end{array}$$

Note that, when a wave-rule is applied, only variables marked as skeleton may be substituted by terms with a skeleton marker!

Reduction of goal with wave-rule (5) via $\{x^s \mapsto x^s\}$:

$$\neg\mathsf{Even}^s(x^s),\ \neg\mathsf{Even}(y),\ \mathsf{Even}^s(\mathsf{s}^{\uparrow}(\mathsf{s}^{\uparrow}(x^s)) +^s y)$$

Wave-rule (4) via $\{x^s \mapsto \mathsf{s}^{\uparrow}(x^s),\ y^s \mapsto y\}$:

$$\neg\mathsf{Even}^s(x^s),\ \neg\mathsf{Even}(y),\ \mathsf{Even}^s(\mathsf{s}^{\uparrow}(\mathsf{s}^{\uparrow}(x^s) +^s y))$$

Wave-rule (4) via $\{x^s \mapsto x^s,\ y^s \mapsto y\}$:

$$\neg\mathsf{Even}^s(x^s),\ \neg\mathsf{Even}(y),\ \mathsf{Even}^s(\mathsf{s}^{\uparrow}(\mathsf{s}^{\uparrow}(x^s +^s y)))$$

Wave-rule (5) via $\{x^s \mapsto x^s +^s y\}$:

$$\neg\mathsf{Even}^s(x^s),\ \neg\mathsf{Even}(y),\ \mathsf{Even}^s(x^s +^s y)$$

Fully rippled! Fertilization via $\{x \mapsto x^s,\ y \mapsto y\}$.

Example 1.3

Given: $\mathsf{revtail}(h, \mathsf{append}(k, l)) = \mathsf{append}(\mathsf{revtail}(h, k), l)$.

Note that all variables in this goal are sinks!

Goal: $\mathsf{revtail}^s(\mathsf{cons}^\uparrow(x, h^s), \mathsf{append}^s(k, l)) = \mathsf{append}^s(\mathsf{revtail}^s(\mathsf{cons}^\uparrow(x, h^s), k), l)$.

This goal is suggested by recursion analysis.

Wave-Rules:

$$\begin{array}{lrcl} (6) & \mathsf{revtail}^s(\mathsf{cons}^\uparrow(x, k^s), l^s) & = & \mathsf{revtail}^s(k^s, \mathsf{cons}^\downarrow(x, l^s)) \\ (7) & \mathsf{cons}^\downarrow(x, \mathsf{append}^s(k^s, l^s)) & = & \mathsf{append}^s(\mathsf{cons}^\downarrow(x, k^s), l^s) \end{array}$$

Note that, when a wave-rule is applied, only variables marked as skeleton may be substituted by terms with a skeleton marker!

Reduction of left-hand side of goal:

$$\mathsf{revtail}^s(\mathsf{cons}^\uparrow(x, h^s), \mathsf{append}^s(k, l))$$

Wave-rule (6) via $\{x \mapsto x,\ \ k^s \mapsto h^s,\ \ l^s \mapsto \mathsf{append}^s(k, l)\}$:

$$\mathsf{revtail}^s(h^s, \mathsf{cons}^\downarrow(x, \mathsf{append}^s(k, l)))$$

Wave-rule (7) via $\{x \mapsto x,\ \ k^s \mapsto k,\ \ l^s \mapsto l\}$:

$$\mathsf{revtail}^s(h^s, \mathsf{append}^s(\mathsf{cons}^\downarrow(x, k), l))$$

Fully rippled! Cross fertilization via $\{h \mapsto h^s,\ \ k \mapsto \mathsf{cons}^\downarrow(x, k),\ \ l \mapsto l\}$ and removal of annotaton

$$\mathsf{append}(\mathsf{revtail}(h, \mathsf{cons}(x, k)), l)$$

Reduction of right-hand side of goal:

$$\mathsf{append}^s(\mathsf{revtail}^s(\mathsf{cons}^\uparrow(x, h^s), k), l)$$

Wave-rule (6) via $\{x \mapsto x,\ \ k^s \mapsto h^s,\ \ l^s \mapsto k\}$:

$$\mathsf{append}^s(\mathsf{revtail}^s(h^s, \mathsf{cons}^\downarrow(x, k)), l)$$

Fully rippled! After removal of annotation, this is equal to reduced form of left-hand side.

Example 1.4

Given: $\mathsf{Perm}(l,l) = \mathsf{true}$.

Note that all variables in this goal are sinks!

Goal: $\mathsf{Perm}^s(\mathsf{cons}^{\uparrow}(x,l^s), \mathsf{cons}^{\uparrow}(x,l^s)) = \mathsf{true}$.

This goal is suggested by recursion analysis.

Wave-Rules:

$$
\begin{array}{lrcll}
(8) & \mathsf{Perm}(\mathsf{cons}^{\uparrow}(x,k^s),l^s) & = & \mathsf{Perm}(k^s,\mathsf{delonce}^{\downarrow}(x,l^s)) & \longleftarrow \quad \mathsf{mbp}(x,l) = \mathsf{true} \\
(9) & \mathsf{Perm}(\mathsf{cons}^{\uparrow}(x,k^s),l^s) & = & \mathsf{false} & \longleftarrow \quad \mathsf{mbp}(x,l) \neq \mathsf{true} \\
(10) & \mathsf{delonce}^{\downarrow}(x,\mathsf{cons}^{\uparrow}(y,l^s)) & = & l^s & \longleftarrow \quad x = y \\
(11) & \mathsf{delonce}^{\downarrow}(x,\mathsf{cons}^{\uparrow}(y,l^s)) & = & \mathsf{cons}^{\uparrow}(y,\mathsf{delonce}^{\downarrow}(x,l^s)) & \longleftarrow \quad x \neq y
\end{array}
$$

Note that, when a wave-rule is applied, only variables marked as skeleton may be substituted by terms with a skeleton marker!

Reduction of goal with wave-rule (8) via $\{x \mapsto x,\ \ k^s \mapsto l^s,\ \ l^s \mapsto \mathsf{cons}^{\uparrow}(x,l^s)\}$:

$$\mathsf{Perm}^s(l^s,\mathsf{delonce}^{\downarrow}(x,\mathsf{cons}^{\uparrow}(x,l^s))) = \mathsf{true},\ \ \mathsf{mbp}(x,\mathsf{cons}(x,l)) \neq \mathsf{true}$$
$$\mathsf{Perm}^s(\mathsf{cons}^{\uparrow}(x,l^s),\mathsf{cons}^{\uparrow}(x,l^s)) = \mathsf{true},\ \ \mathsf{mbp}(x,\mathsf{cons}(x,l)) = \mathsf{true}$$

First clause is reduced with wave-rule (10) via $\{x \mapsto x,\ \ y \mapsto x,\ \ l^s \mapsto l^s\}$:

$$\mathsf{Perm}^s(l^s,l^s) = \mathsf{true},\ \ x \neq x,\ \ \mathsf{mbp}(x,\mathsf{cons}(x,l)) \neq \mathsf{true}$$
$$\mathsf{Perm}^s(l^s,\mathsf{delonce}^{\downarrow}(x,\mathsf{cons}^{\uparrow}(x,l^s))) = \mathsf{true},\ \ x = x,\ \ \mathsf{mbp}(x,\mathsf{cons}(x,l)) \neq \mathsf{true}$$

The first clause is fully rippled and tautology by fertilization. The second clause is a reflexive tautology.

The remaining clause from above can be reduced with wave-rule (9) via $\{x \mapsto x,$ $k^s \mapsto l^s,\ \ l^s \mapsto \mathsf{cons}^{\uparrow}(x,l^s)\}$:

$$\mathsf{false} = \mathsf{true},\ \ \mathsf{mbp}(x,\mathsf{cons}(x,l)) = \mathsf{true}$$

This simplifies to

$$\mathsf{mbp}(x,\mathsf{cons}(x,l)) = \mathsf{true}$$

and then to the reflexive tautologies

$$\mathsf{true} = \mathsf{true},\ \ x \neq x$$
$$\mathsf{mbp}(x,l) = \mathsf{true},\ \ x = x$$