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Book notice

David Hilbert and Paul Bernays: Grundlagen der mathematik I. Foundations of mathematics I. Part A. Edited by Wirth, Claus-Peter, Jörg Siekmann, Michael Gabbay, and Don Gabbay. Commented translation by Claus-Peter Wirth. London: College Publications, 2011, lxiii+xv+44pp.+Appendix, £14.50 PB

Vassilis Sakellariou

The first bilingual German–English edition of Hilbert's and Bernays' seminal work, 'Grundlagen der Mathematik', is a significant addition to the arsenal of historians and philosophers of mathematics in general, and researchers in foundational issues in particular. It fills a bibliographical gap by making this trailblazing work accessible and thus helping to properly understand the origins of metamathematics. The volume at hand is Part A of Volume I and presents, as the editors point out, 'the motivation and philosophical foundation' of the authors' 'original view on finitistic mathematics and their methodological standpoint for proof theory'. Apart from the main text, it includes prefaces and a republication of an essay on Hilbert's proof theory by Wilfried Sieg. The editors, in their preface, place Hilbert's and his collaborators' work in its historical context and provide a broad outline of their programme aiming at proving the consistency of mathematics. Sieg's essay constitutes in itself an insightful and stimulating contribution. Based on unpublished source material, it traces the 'circuitous path from Hilbert's logicist position […] to the novel program of finitistic proof theory' and beyond. The translation of the original text's rich and philosophically sophisticated language has been a formidable task demanding considerable erudition. Claus-Peter Wirth rose masterfully up to the challenge. Annotations and numerous notes, well researched and documented, complement the main text and bring out its multifaceted character, assisting the reader to appreciate its full scope and to reflect on it. The continuous support of a number of distinguished scholars made the edition possible and contributed to its high quality.

Part A comprises §§1 and 2 of the whole work. In §1, the authors introduce their 'sharpened' axiomatic standpoint as the appropriate treatment of the central task posed by the state of research in the foundations of mathematics: to deal with the problem of the infinite in a new way. In this light, the distinction is made between what the authors

V. Sakellariou (✉)
Department of Philosophy and History of Science, University of Athens, University Campus, Athens 15771, Greece
e-mail: sakellariouv@phs.uoa.gr

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call the ‘constructive or genetic’ method and their own axiomatic method, understood in a narrow sense and involving the so-called ‘existential form’ as an additional factor. It is this ‘formal axiomatics’ that requires a proof of consistency which can only be an impossibility proof. This section concludes with the question whether such a proof can be given without any axiomatic-existential assumptions, and so ground the whole of arithmetic and, therefore, all axiomatic theories which can be reduced to it.

In addressing this question, §2 introduces a novel kind of inference which the authors call ‘finitistic’. However, adopting the finitistic standpoint does not mean that the question of consistency of non-finitistic methods in arithmetic is rendered irrelevant. On the contrary, the authors point to the ‘considerable loss of systematics and proof techniques’ in substantial areas of mathematics if one insists, as the adherents of the school of intuitionism did, on avoiding non-finitistic methods altogether. Hilbert and Bernays are not prepared to follow this path. This means that while they adopt the finitistic standpoint, they nevertheless have to confront the ‘problem of obtaining a clear insight into the applicability of non-finitistic methods’, which amounts to ‘gaining certainty that these customary methods of arithmetic can never lead to a demonstrably false result’. Hilbert and Bernays admit that they are far from solving this problem but, as they point out, their work has opened up a new field, the systematic proof theory or metamathematics.

One can only hope that this valuable work will be completed with the publication of both volumes of the original German.