than p' and not the sum of two squares and so on, until reaching 5, a number that, according to the construction, should not be the sum of two squares. But, as 5 is the sum of two squares, this implies an absurdity that arises from supposing p not to be sum of two squares, hence p is the sum of two squares.

More general: Let us suppose that we must prove the theorem $\forall x.\ T(x)$. Suppose that $T(x)$ is true for x among some initial values. It is possible to use the expression "$T(x)$ is true for x being a small number". For S being the predicate that holds exactly for these small numbers, we then have $\forall x.\ (S(x) \Rightarrow T(x))$. Now let us suppose that $T(n)$ is false for an arbitrary natural number n, and that we are able to construct an algorithm such that, if $T(n)$ is false, then $T(m)$ is false for a number m smaller than n. (This description lacks generality: „algorithm" means computability. But we do not even need constructiveness, not even describability, we just need existence.)

(The following question is most serious imho: Regarding the actual theorem proving activity of the mathematician, this method of reduction descent differs from the method of indefinite descent only if he may additionally assume that n is not a small number. So, please, Poalo, answer the following question to me: Does $\left(\begin{array}{c} \neg S(n) \wedge \neg T(n) \\ \Rightarrow \quad \exists m \prec n.\ \neg T(m) \end{array}\right)$ suffice or is the mathematician required to show $\left(\begin{array}{c} \neg T(n) \\ \Rightarrow \quad \exists m \prec n.\ \neg T(m) \end{array}\right)$ for the method of reduction descent actually? In the later case I would be very unhappy for two reasons:

1. I did not understand you and your book properly up to now and what I wrote about you in SWP–2006–02 is wrong although you counter-checked it. It is just now in the printing factory, and I would like to stop it if possible and if to prove the former would not suffice for the method of reduction descent.
2. I do not think the distinguishing of the two concepts „indefinite descent" and „reduction descent" to be appropriate *in any mathematical sense, even not in the historical one*, but judge this as a sophistication which is „over the top" similar to what I wrote in §2.4.1 on Unguru and Acerbi in SWP–2006–02. Sorry for being so horribly explicit, but I am a German, after all, even if I do not like the Germans.

Even if you think that the latter is actually required, maybe Fermat thought differently? What do the many reconstructions of Sergio Paolini say about this? Is there any example of a reduction descent where $\neg S(n)$ is used in the proof, i.e. the reduction assumes that then n is not a small number? A single example would show that the former alternative is right and that we all could be happy!) Let us suppose that the process can be iterated and that the "small numbers" are reached. Obviously T might be false in particular exactly for the "small numbers". However, for the small numbers T is true, so we have a contradiction and such a contradiction arises because we have supposed $\neg T(n)$ to be true. Therefore T is true for every number. We will call this method *reduction-descent*. Thus, in the set of methods which can be called descent, it is possible to distinguish, following Fermat's implicit indications:

1. the indefinite descent in a proper sense where if we suppose false the theorem to prove we have:
   (a) a number m, less than n;
   (b) a reduction that starts from n and that, from a formal point of view, can be continued infinitely, but that cannot reach m for particular mathematical reasons which are specified in every single theorem;
   (c) the absurdity which derives from this situation: an infinite number of integers should subsist between m and n;